

# Few- and many-body methods in nuclear physics

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**Abstract.** In this contribution a brief overview of the status and perspectives of the theoretical methods for studying light and heavy nuclear systems is presented.

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## 1 Introduction

Current nuclear-physics research includes a very rich variety of phenomena. The structure of hadrons and nuclei, the properties of the different phases of nuclear matter, the origin of the elements through primordial and stellar nucleosynthesis, the test of fundamental symmetries in nuclei represent very broadly the main themes and questions in nuclear science today [1, 2]. In this contribution, we shall concentrate on the primary goal of nuclear physics, namely the study of the structure and dynamics of nuclear systems. Some of the main questions in this case are the following: 1) How are complex nuclei built from their constituents? 2) How well can nuclear reaction rates be computed? Etc.

The “standard” starting point in trying to answer these questions is to consider the nucleus as composed of nucleons interacting via non-relativistic potentials mediated by pions and other mesons. In this model there are also several issues still not well established, in spite of decades of studies. For example, the nature of the nucleon-nucleon (NN) and of the three-nucleon (3N) interactions, the importance of many-body (correlation) effects with respect to single-particle properties of the nuclear medium and the limits of the nuclear stability, namely the so-called neutron and proton drip lines, etc., are still open problems.

Moreover, in recent years new and relevant developments have taken place. In fact, whereas only the stable nuclei could be studied until a few years ago, nowadays it is possible to observe and perform measurements of the specific properties (still with some limitations) of a large number of unstable nuclei of short lifetime. Most of the properties of these new nuclei are somewhat different from those already known. For example, they have a large “halo”, as seen from their size which has been found to be larger than that given by the standard rule  $R = r_0 A^{1/3}$ .

Also the shell structure of the nuclei around the drip lines has offered some surprises.

The remarkable advances in the treatment of the nuclear many-body problem have greatly improved our understanding of various nuclear systems (as well as the technical developments of accelerators, detectors and data analysis techniques). In this contribution a brief overview of the status and perspectives of the theoretical methods used to study light and heavy nuclear systems is presented. Clearly, due to the limited space, we have selected a few, illustrative topics. First of all, a detailed discussion of our current understanding of the nuclear interaction is reported in sect. 2. Some important recent advances in the study of the medium-heavy and light systems are examined in sects. 3 and 4, respectively. Finally, the conclusions are given in sect. 5.

## 2 The nuclear interaction

The starting point of “standard” nuclear physics is the interaction between nucleons. It is a rather complicated problem and a well-founded theory has not yet been achieved. The NN interaction is known very well only for large interparticle separations where the one-pion exchange potential (OPEP) is the dominant term. At shorter distances the situation is rather complicated. Usually one resorts to NN scattering experiments to obtain valid parameterizations in the inner regions. In the next three subsections, the different approaches used to parameterize the NN interaction are summarized. Finally, in the last subsection, a brief discussion of the current understanding of the 3N force is reported.

### 2.1 Phenomenological models

About two decades ago, the Nijmegen group started a program to improve the phase shift analysis (PSA) of NN

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scattering data below 350 MeV [3]. In their final result, the Nijmegen PSA, they consider all the NN data taken from 1955 to 1992 and reject the points with improbably high  $\chi^2$  (about 30% of the data was disregarded). The remaining data could be fitted with an energy-dependent PSA [3] having a  $\chi^2/\text{datum}$  of 0.99.

Based on this “pruned” database, in the period 1994–2001 a new class of charge-dependent NN potentials was constructed. The new potentials constructed were: 1) the Nijmegen potentials (Nijm-I, Nijm-II and Reid93) [4]; 2) the Argonne potential (AV18) [5]; 3) the Bonn potential (CD Bonn) [6]; and 4) the non-local potentials of Doleschall and collaborators [7]. All these potentials have in common the use of approximately 45 parameters and fit the Nijmegen data set with  $\chi^2/\text{datum} \approx 1$ . These modern potentials use different short- and intermediate-range parameterizations (they have all in common the long-range OPEP). Moreover, they differ in other properties, for example the CD Bonn potential is non-local in coordinate space while AV18 is local.

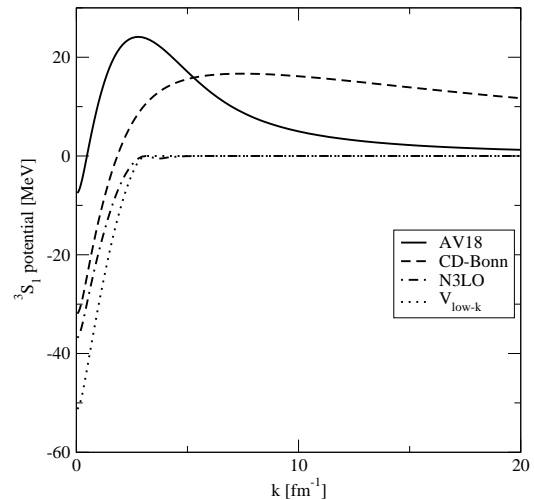
## 2.2 Nuclear forces from effective field theories

Alternatively, one can derive NN forces from an effective field theory (EFT) starting from a chiral Lagrangian. In this approach, which was pioneered by Weinberg [8], one starts from a Lagrangian describing pions and nucleons which takes into account the (approximate) chiral symmetry. Such Lagrangians reproduce correctly the physics of  $\pi N$  scattering at low energies [9]. The central point of this approach is the definition of a power counting scheme which permits a systematic truncation of the possible contributions to the interaction (chiral perturbation theory).

The most serious attempts to construct an NN interaction from an EFT have been performed in refs. [10] and [11], where potentials have been developed to the next-to-next-to-next-to-leading-order (N<sup>3</sup>LO) in the chiral expansion. In their current form, these potentials contains 27 parameters (coming from the unknown coupling constants of the contact terms in the chiral Lagrangian). These parameters have been fitted to the NN database, which nowadays has been noticeably enriched with respect to the Nijmegen database, with a  $\chi^2$  very close to 1 (the best result has been achieved for the so-called N3LO potential of ref. [11]). Therefore, these chiral potentials have reached an accuracy comparable to those achieved for the phenomenological potentials discussed in sect. 2.1.

## 2.3 Effective forces

The standard description of the NN interaction emerging from the approaches described above is a short-range repulsion and a longer-range attraction. In recent years however, it has been argued that, if one is interested in low-energy observables, a low-momentum interaction could be used [12,13]. This potential  $V_{low-k}$  can be derived from one of the “bare” NN interactions discussed previously by “eliminating” the high-momentum part for



**Fig. 1.** Diagonal part of the  ${}^3S_1$  potential in momentum space for the AV18 [5], CD Bonn [6], N3LO [11] and one version of the  $V_{low-k}$  [13] interaction models.

$k > \Lambda \approx 2 \text{ fm}^{-1}$  using renormalization group techniques or with a unitary transformation. This approach has a number of attractive features. First of all, for  $k < \Lambda$ ,  $V_{low-k}$  reproduces all the NN observables (including the deuteron properties). Secondly, it is universal, in the sense that starting from different bare interactions an essentially unique  $V_{low-k}$  is derived. Finally, it can allow for shell model calculations in heavy nuclei (see sect. 3.2). Similar approaches have been proposed in refs. [14] and [15].

To give an idea of the behavior of these interactions, the on-shell part of some of these potentials in momentum space is plotted in fig. 1. As can be seen from the figure, the “phenomenological” AV18 and CD Bonn potentials have large high-momentum tails, coming from the strong repulsion at short interparticle distances. The N3LO potential is much softer (it vanishes for  $k > 5 \text{ fm}^{-1}$ ). Finally, the  $V_{low-k}$  potential is even softer (it does not contain any repulsion in coordinate space).

## 2.4 The 3N forces

In the study of systems with  $A \geq 3$ , one should also add appropriate many-nucleon forces. It is currently believed that only the 3N forces should play a relevant role in light systems (for a first quantitative estimate of the magnitude of a four-nucleon force, see ref. [16]). Several models of 3N forces have been proposed, mainly based on the exchange of pions among the three nucleons. The best known of these models are the Tucson-Melbourne [17] (TM) and the Brazil [18] (BR) 3N forces. Another model, the Urbana [19] (UR) 3N force is based on the Fujita-Miyazawa mechanism (exchange of two pions with the excitation of a  $\Delta$ ). Other models are discussed, for example, in ref. [20].

New mechanisms for the 3N force have also been proposed using the chiral approach [21]. The development of

a 3N force at N<sup>3</sup>LO is currently under way [22]. In this approach, it is possible to prove that the 3N forces compare at a large order (N<sup>2</sup>LO) in the chiral power counting; their effects are thus suppressed compared to NN interactions [23].

The Urbana-Argonne group has in recent years developed [19,24] a formidable technique, the Green Function Monte Carlo (GFMC) method (see below), which can be successfully applied to calculate the spectra of nuclei up to  $A = 12$ . They have shown that using the AV18 NN potential + UR 3N force (version IX) one does not obtain a good description of the spectra of such nuclei. For this reason a new model of 3N interaction has been proposed [25], the Illinois 3N force, incorporating three pion exchange “box” diagrams. Using this model together with the NN AV18 potential, a good description of the full spectrum of the light nuclei with  $A \leq 12$  has been reached. However, this potential model contains some terms which do not respect the chiral counting order, and for this reason it is not equivalent with the models developed using the chiral approach. This issue is still under debate.

As we have seen, several new models of NN and 3N interactions have recently been developed. It is therefore of interest to perform tests in order to understand which of these different models should be more appropriate for describing the very complex nature of the various nuclear systems.

### 3 Many-body methods

To study the structure of medium to heavy nuclei a large variety of techniques has been used. Some of them are discussed briefly in the following subsections.

#### 3.1 Microscopic approaches

One of the traditional approaches for studying the nuclear structure is to try to derive the properties of the nuclei from the “bare” interaction between nucleons, as derived from the PSA of NN data (microscopic approach), by taking the strong repulsion at short interparticle distances fully into account. To be successful, one has to take into account a proper 3N force in these approaches.

The best-known solution to this problem lies in the Brückner theory [26,27]. Nowadays, nuclear and neutron matter calculations have been performed up to three hole lines diagrams and extended up to rather high densities, of the order of 6 times the saturation density. The calculated equation of state (EOS), namely the binding energy per nucleon in function of the density, for high density is a critical quantity necessary for understanding the properties of compact stars. Spin- and isospin-polarized nuclear-matter calculations have also been performed [28].

One of the key points of a calculation of the EOS is the reproduction of the empirical saturation point of nuclear matter (namely, the binding energy per particle  $B/A$  of nuclear matter should have a maximum of 15 MeV at the empirical density  $\rho_0 = 0.16 \text{ fm}^{-3}$ ). It is well known that for

obtaining that value with a local potential like the AV18 potential, it is essential to include a strong repulsive core and a proper 3N force [27]. Note that for reproducing the empirical saturation point, one has to change the parameters of the 3N force, whose values therefore differ from those derived in the  $A = 3, 4$  systems [29].

Recently, the effect of using non-local potentials has been investigated. The cases of CD Bonn and Doleschall potentials have been studied in refs. [30,31] and ref. [32], respectively. In both cases, the value of  $B/A$  is found to be rather larger than the empirical one. This indicates that to get the correct saturation point, one needs an amount of 3N force larger than usual.

One of the key problems is to know the uncertainties of the computed EOS. From this point of view, the comparison with the EOS calculated using other techniques is very helpful. Another approach used extensively for studying nuclear matter and medium-heavy nuclei starting directly from the bare NN interaction is the Correlated Basis Function (CBF) theory [33]. In this approach, one starts with a basis of states where the correlations are already built-in. It is therefore a method very well adapted to treating cases where the interaction has a strong repulsion at short range.

A detailed comparison between the Brückner and CBF results has been performed recently in ref. [34]. The results are found to be in agreement up to densities  $\sim 1.5\rho_0$ . For larger densities the differences may become rather sizeable especially in the presence of a 3N force. Further studies will be necessary to clarify this discrepancy.

The Brückner and CBF theory have also been used for making calculations in finite systems [35–38] with good results. An application where the Brückner theory is used to calculate the rate of the double beta decay is reported in ref. [39].

As already mentioned, another powerful approach for studying the structure of ( $A \leq 12$ ) nuclei is the GFMC technique developed by the Urbana group [19,24]. It would be interesting to apply this model also to the study of the medium-heavy nuclei and nuclear matter. Some calculations in this direction have been performed in ref. [40]. However, the application of the GFMC technique becomes more and more time consuming for  $A > 13$  due to the large number of spin-isospin states.

A way of overcoming this problem has been proposed in ref. [41]. This method, the so-called auxiliary field diffusion Monte Carlo (AFDMC), is still based on a Monte Carlo technique. The summation over the spin-isospin degrees of freedom is simplified and this allows for the extensions to larger systems. Applications has been presented for nuclear and neutron matter, small neutron droplets and medium-heavy nuclei [41,42].

#### 3.2 Effective approaches

As discussed in sect. 2.2, a number of “effective” interactions have recently been devised, where the repulsive part has been eliminated. Thus, these “effective” potentials can

be used in shell model calculations without the need to use the Brückner resummation.

A study of nuclear matter using the  $V_{low-k}$  potential has been performed in ref. [43]. In this case, also a TM-like 3N force has been included in the calculation (note that the parameters of the 3N force were chosen to be the same as determined from the  ${}^3\text{H}$  and  ${}^4\text{He}$  binding energies [44]). The EOS, computed in the Hartree-Fock approximation plus dominant second-order corrections of the standard perturbation theory, is found to be rather realistic. The computed saturation point is close to the empirical value. A more complete calculation is currently underway.

Several shell model calculations for finite nuclei have also been performed. For example, with the  $V_{low-k}$  potential, the binding energies and radii of the closed-shell nuclei  ${}^4\text{He}$ ,  ${}^{16}\text{O}$ , and  ${}^{40}\text{Ca}$  have been found to be in agreement with the experimental data for less than 1%, and that the theoretical findings depend only slightly on the bare NN potential used as starting point [45,46]. Other shell model calculations have been performed, for instance for  ${}^{100}\text{Sn}$  and  ${}^{132}\text{Sn}$ . These, respectively, proton- and neutron-rich nuclei are unstable and, as mentioned in the introduction, understanding the shell structure far from stability is one of the new frontiers of nuclear physics. In general, the result of shell model calculations are able to reproduce well the experimental data for the low-lying levels [47].

Analogously, the potential derived in ref. [14] using the unitary correlation operator method (UCOM) has been used for studying a large number of nuclei. A remarkable agreement of the calculated ground-state energies has been found over the whole mass range from  ${}^4\text{He}$  to  ${}^{208}\text{Pb}$  [48].

### 3.3 Other approaches

For reasons of space, a number of other approaches have to be ignored. Amongst them, we can mention the use of simpler effective forces (like the Skyrme and Gogny forces), the in-medium chiral perturbation theory approach, the various algebraic methods (such as the interacting boson model), the energy functional methods, and many others. For a general review, see, for example, ref. [49].

## 4 Few-body methods

Historically, the study of the nuclear interaction has been the central goal in few-nucleon systems. In fact, it is possible to achieve numerically accurate solutions of the associated quantum-mechanical problem, allowing for unambiguous comparisons between theory and experiments [50]. Also in this case, the lack of space forces us to select a few topics. This section is organized as follows: in sect. 4.1, a brief description of the theoretical methods which are applied most to studying few-body systems are briefly recalled. The results obtained for the  $A = 3$  bound states are reported in sect. 4.2. In sect. 4.3, a discussion of the  $N - d$   $A_y$  “puzzle” is presented. Finally, a brief discussion of the reaction  $d+d \rightarrow {}^4\text{He}+\pi^0$  is reported in sect. 4.4.

### 4.1 Theoretical methods

A great deal of effort has been dedicated to the development of techniques for the numerical solution of the non-relativistic Schrödinger equation,

$$H\Psi(1, 2, \dots, A) = E\Psi(1, 2, \dots, A), \quad (1)$$

where  $H$  is a nuclear Hamiltonian. In this section, we will limit ourselves to studying  $A = 3, 4$  systems. In the Faddeev equation (FE) approach [51–54], eq. (1) is transformed to a set of coupled equations for the Faddeev amplitudes, which are then solved directly (in momentum or coordinate space) after a partial wave expansion.

In the GFMC method already mentioned, one computes  $\exp(-\tau H)\Phi(1, 2, \dots, A)$ , where  $\Phi(1, 2, \dots, A)$  is a trial wave function, using a stochastic procedure to obtain, in the limit of large  $\tau$ , the exact ground-state wave function  $\Psi$  [19,24].

The stochastic variational method (SVM) [55,56] and the coupled rearrangement channel Gaussian method (CRCG) [57,58] provide a variational solution of eq. (1) by expanding the (radial part of the) wave function in Gaussians. Another widely applied variational method is the expansion of the wave function using the hyperspherical harmonic (HH) basis [59,60].

Very recently two other new techniques have been proposed. In the no-core shell model (NCSM) method [61,62] the calculations are performed using a (translationally invariant) harmonic-oscillator (HO) finite basis  $P$  and introducing an effective  $P$ -dependent Hamiltonian  $H_P$  to replace  $H$  in eq. (1). The operator  $H_P$  is constructed so that the solution of the equation  $H_P\Psi(P) = E_P\Psi(P)$  provides eigenvalues which quickly converge to the exact ones as  $P$  is enlarged. The effective interaction hyperspherical harmonic (EIHH) method [63,64] is based on a similar idea, but the finite basis  $P$  is constructed in terms of the HH functions.

### 4.2 Bound states of the $A = 3$ system

All of the methods discussed above provide very accurate results for the  $A = 3, 4$  bound states [65]. For example, results for a few representative potential models are reported in table 1 for the  ${}^3\text{H}$  binding energy and other properties. The NN potentials considered are the CD Bonn, N3LO and AV18 potential models.

There is good agreement, at the level of 0.1%, between the results obtained with different techniques for all the quantities considered in the tables. A similar precision is nowadays also achieved for the  $A = 4$  nucleus.

### 4.3 The puzzle of the $N - d$ vector analyzing powers

The  $N - d$  elastic scattering process has been studied extensively in recent years. Most of the calculations have been performed using the “phenomenological” potentials described in sect. 2.1, with the eventual inclusion of the

**Table 1.** The triton binding energies  $B$  (MeV), the expectation values of the kinetic energy operator  $\langle T \rangle$  (MeV), the mean square radii  $\sqrt{\langle r^2 \rangle}$  (fm), and the  $P$  and  $D$  probabilities (all in %), calculated with the CD Bonn, N3LO, and AV18 potentials. The results obtained with various techniques have been reported.

Interaction Method	$B$	$\langle T \rangle$	$\sqrt{\langle r^2 \rangle}$	$P_P$	$P_D$	
CD Bonn	HH [66]	7.998	37.630	1.721	0.047	7.02
	FE [67]	7.997	37.620	-	0.047	7.02
	FE [68]	7.998	37.627	-	0.047	7.02
	NCSM [69]	7.99(1)	-	-	-	-
N3LO	HH [66]	7.854	34.555	1.758	0.037	6.31
	FE [67]	7.854	34.546	-	0.037	6.32
	FE [68]	7.854	34.547	-	0.037	6.32
	NCSM [69]	7.85(1)	-	-	-	-
AV18	HH [70]	7.618	46.707	1.770	0.066	8.511
	FE [67]	7.621	46.73	-	0.066	8.510

TM, BR or UR 3N force. The agreement between these calculations and the large amount of experimental data is in general impressive, except for the vector analyzing power observables. This is a fairly old problem, already reported about 20 years ago [71,72] in the case of  $n-d$  and later confirmed also in the  $p-d$  case [73]. It consists of the fact that all the theoretical calculations based on phenomenological NN potentials (even including TM, BR and UR 3N forces) underestimate the measured nucleon vector analyzing power  $A_y$  by about 20–30%. The same problem also occurs for the vector analyzing power of the deuteron  $iT_{11}$  [73], while the deuteron tensor analyzing powers  $T_{20}$ ,  $T_{21}$  and  $T_{22}$  are reasonably well described [51].

In most of the calculations, the electromagnetic (EM) interaction is usually approximated with the point Coulomb potential (sometime also this term is disregarded). Given the extreme sensitivity of the observables  $A_y$  and  $iT_{11}$  to the spin-orbit interaction [74,75], it has been recently investigated [76,77] whether this problem

could be related to the absence of the magnetic-moment (MM) interaction between the nucleons in the theoretical studies. The results obtained in ref. [76] for  $A_y$  at  $E_{lab} = 1$  and 3 MeV for  $p-d$  elastic scattering have been reported in fig. 2. As can be seen, the effect of the MM interaction is quite sizable, but insufficient to solve the disagreement with the data.

The calculation of  $A_y$  with the chiral potentials is still under way. Preliminary results have shown that the situation is unclear as this observable depends rather strongly on the the cutoff  $\Lambda$  of the theory [80]. Calculation with the effective interactions are still to be performed.

The attention has also been focused on exotic 3N force terms not contemplated so far [81,82]. Calculations are under way to see the effect of the Illinois 3N force and the chiral 3N force at the N<sup>3</sup>LO level. In the next few years, there is hope of solving this puzzle.

Similar discrepancies have also been observed in the four-nucleon system [83]. As an example, the experimental total  $n-^3\text{H}$  cross-sections for  $E_n = 3-5$  MeV have been found at variance with all the theoretical calculations performed so far [84].

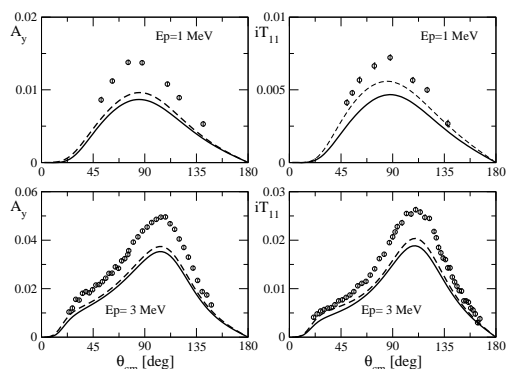
#### 4.4 The reaction $d + d \rightarrow \alpha + \pi^0$

The good accuracy which can nowadays be obtained in the numerical solution of four-nucleon problems allows for interesting studies and applications. Among them, the study of charge symmetry breaking (CSB) terms in the NN interaction by means of the  $d + d \rightarrow \alpha + \pi^0$  reaction is rather appealing. The CSB terms in the nuclear interaction come from both the  $u$ - and  $d$ -quark mass difference (a fundamental quantity poorly known) and from EM effects. The recent observation of the  $d + d \rightarrow \alpha + \pi^0$  reaction at IUCF [85] is a direct probe of CSB. The theoretical study of this reaction is currently under way [86].

## 5 Conclusions

In recent years, there have been important developments in the field of many- and few-nucleon systems. Here, we have concentrated our attention mainly on studying the infinite nuclear matter, heavy nuclei and few-nucleon systems. The link between these fields lies in the use of the same interaction models to describe their properties. The hope is to provide a “unified” picture of all the nuclear systems starting from the microscopic NN + 3N interaction.

An important point to be stressed is that nuclear physics is embedded into the context of other sciences. A large part of problems in nuclear and atomic/molecular physics are nowadays tackled using the same methods. The few-body techniques described here are also currently used for studying the structure of hadrons in terms of constituent quarks. Studies in nuclear physics have also a big impact in particle physics, cosmology and astrophysics. In fact, the linkage between these fields is fundamental for our understanding of the base structure of our universe. The increase of the capability to provide model-independent predictions of nuclear structure and reactions will contribute strongly to the knowledge of our world.



**Fig. 2.** The  $p-d$   $A_y$  and  $iT_{11}$  calculated using AV18 (solid lines) and AV18 + MM (dashed lines). Experimental points are from ref. [78] (1 MeV) and ref. [79] (3 MeV).

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